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Aerostructural Safety Factor Criteria Using Deterministic Reliability

Vincent Verderaime*
NASA Marshall Space Flight Center, Huntsville, Alabama 35812

The primary function of most metallic structures is to sustain operational loads with no detrimental deformation. The current practice of specifying a universal ultimate safety factor, or reliability, biases the operation margin by the selection of material. By expressing the deterministic safety factor in probabilistic tolerance limits, the included three safety design factors are specified independently by loads, materials, and stress criteria. The minimum limit of the stress safety design factor was determined to be the ultimate-to-yield stress ratio. This ratio was combined with estimated modeling and complexity uncertainty factors to redefine the conventional safety factor. The metallic structural safety with the modified conventional safety factor and the loads and material safety design factors was expressed by a safety index and reliability. Although the resulting reliability varies with material selection, the suggested criteria provide a uniformly efficient and safe operational structure.

Nomenclature

C = coefficient of variation, σ/μ
F = deterministic single valued stress
F_{tu} = ultimate tensile stress
F_{ty} = yield tensile stress
K = sample size tolerance factor
m = structural weight coefficient
N = statistical sample size
n = probabilistic range factor
P = probability
q = well-defined mixture fraction
R = reliability
SF = conventional safety factor
SF_{LL} = lean safety factor
w = weight
Z = safety index
λ = safety factor zone
μ = mean (expected value)
σ = standard deviation

Subscripts

A = applied stress
$A1$ = well-defined applied stress
$A2$ = less well-defined applied stress
R = resistive stress
tu = ultimate stress
ty = yield stress

Introduction

CURRENT structural safety design practices are considered inadequate for future launch vehicles and spacecraft. The deterministic method has consistently dominated the industry for over 50 years with limited improvement in providing efficient operational structures. This method assumes that the material strength and the load-induced stress are single valued (constants). Their ratio is an arbitrarily specified ultimate safety factor to account for design uncertainties. It is universally applicable to most structural problems, and it is verifiable. However, the method is not generally understood, and it is perceived as arbitrarily applied and too conservative, although it has been related to 30% of development structural failures.

The probabilistic method considers the uncertain nature of measured material strength data and predicted load-induced stresses as random variables with assumed probability distributions. Failure occurs when the load-induced stress distribution exceeds the material strength distribution. It provides realism through rigorous analyses of detailed environments and design parameters through statistical and probabilistic techniques. Recent developments in probability methodology, as well as increasing capacity of digital computers, have substantially advanced its design suitability. It is the most funded and researched technique. Although it promises to be a superior method, it currently is evolving as too labor and data intensive, contractually unverifiable, and incompatible with design room dynamics.

This article incorporates the implied probability features of the deterministic concept to establish safety design factors for defining first-order structural reliabilities. Safety design factors are identified and selected on a coherent philosophy of risk and structural efficiency. Normal probability distributions were assumed throughout this study, although lognormals are applicable and recommended for distributions having coefficients of variation in excess of 15%.

Deterministic Safety Concept

The conventional safety factor defined for static stress is a numerical value by which the product of the conventional safety factor and the maximum expected applied stress does not exceed the minimum ultimate strength of the structural material,

$$SF \times F_A = F_R \tag{1}$$

Maximum and minimum stated limits imply a statistical range of variations about their expected values[1] that may be expressed by upper and lower tolerance limits[2] of their normal stress distributions,

$$F_A = \mu_A + n_A\sigma_A \tag{2}$$

$$F_R = \mu_R - n_R\sigma_R \tag{3a}$$

or

$$F_R = SF \times (\mu_A + n_A\sigma_A) \tag{3b}$$

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*Aerospace Engineer, Structures and Dynamics Laboratory.

The safety factor concept and properties expressed in Eqs. (1–3) are illustrated in Fig. 1, where the distributions are defined by probability density functions. Their overlapping tails suggest the probability that a weak resistive material will encounter an excessively applied stress to cause failure. The overlap is governed by the difference of the two distribution means μ_R - μ_A. The content of this difference is the focus of the deterministic reliability concept.

The difference of the distribution means is composed of three zones: two tolerance limits, defined by Eqs. (2) and (3a), and a midzone λ. Each zone is noted to incorporate a safety design factor that is independently selected to specify an explicit zone range requirement. The probability range factor of the ultimate resistive stress is specified by the probability and confidence required of the material strength $F_R=F_{tu}$. The probability range factor of the applied stress is specified by the desired probability that stresses induced by operational design loads will not exceed the material yield property, i.e., $F_A<F_{ty}$. The midzone is defined by the difference of Eqs. (2) and (3b),

$$\lambda = SF\times(\mu_A+n_A\sigma_A)-\mu_A-n_A\sigma_A = (SF-1)\times(\mu_A+n_A\sigma_A) \tag{4}$$

which is governed by an unbounded conventional safety factor. The problem is to determine its physical limits or pressing requirements.

Letting the safety factor equal unity, the midzone vanishes, making the maximum applied stress coincide with the ultimate resistive stress $F_R=F_A=F_{tu}$. This is an unacceptable condition for polycrystalline materials because the applied stress must then operate in the inelastic region through deformation and fracture. A condition for which the applied stress does not yield the polycrystalline material is $F_A=F_{ty}$, which is substituted into Eq. (1) to establish the lower limit of the conventional safety factor, here coined as the lean safety factor,

$$SF\geq SF_{LL}=\frac{F_{tu}}{F_{ty}} \tag{5}$$

Combining all three zones of Fig. 1 reduces the difference of the distribution means to

$$\mu_R-\mu_A=\mu_A(SF-1)+SF(n_A\sigma_A)+n_R\sigma_R \tag{6}$$

which is a function of all three safety design factors (n_A, n_R, and SF). The net significance of these combined design factors on the interference of applied and resistive stress distribution tails is derived from reliability principles.

Deterministic Reliability

The resistive stress in Fig. 1 defines the uniaxial strength data distribution. The applied stress distribution of load-induced multiaxial stresses is converted into von Mises stress. Given that both

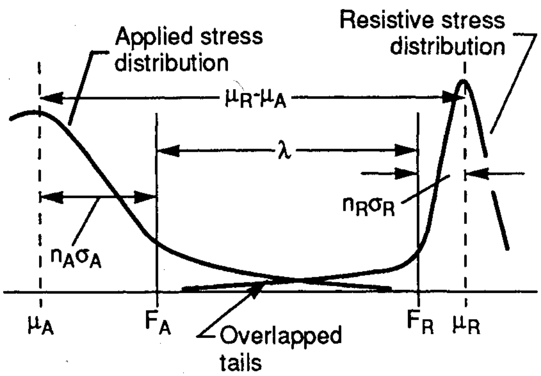

Fig. 1 Deterministic safety concept.

Table 1 Reliability sensitivity to variations in safety design factors

Example	SF	n_R	n_A	μ_A	Z_{tu}	R_{tu}	Z_{ty}	R_{ty}
1	1.25	2.4	2	37.7	4.34	0.9_5	2.46	0.9_2
2	1.25	3.3	2	36.8	4.56	0.9_6	2.67	0.9_2
3	1.25	3.3	3	32.5	5.83	0.9_8	3.72	0.9_4
4	1.40	3.3	4[a]	29.0	7.11	0.9_{11}	4.77	0.9_6
5	1.40	3.3	3[a]	29.0	7.11	0.9_{11}	2.97	0.9_3

[a]Effective probability range factor.

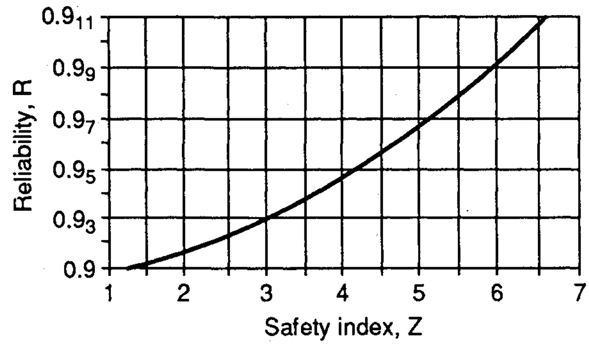

Fig. 2 Reliability vs safety index.

probability density functions are normal and independent, they may be combined to form a third random variable density function[3] through the convolution of two normals resulting in the expression

$$Z=\frac{\mu_R-\mu_A}{\sqrt{\sigma_R^2+\sigma_A^2}} \tag{7}$$

which is known as the safety index. The relationship between the safety index Z and reliability R is given by

$$R=P(F_R-F_A>0)=\Phi(Z)$$

where $\Phi(Z)$ is the standard cumulative normal distribution. Figure 2 is a plot of this relationship.

Substituting the difference of the stress means defined by Eq. (6) into the difference of the means denoted in Eq. (7), the deterministic ultimate safety index is obtained as

$$Z_{tu}=\frac{(SF-1)+SF(n_AC_A)+(n_RC_R)\mu_R/\mu_A}{[C_A^2+(C_R\mu_R/\mu_A)^2]^{1/2}} \tag{8}$$

In calculating the safety index from Eq. (8), the ultimate stress reliability is determined from Fig. 2. Insight into the sensitivity of the deterministic reliability to variations in safety design factors may be demonstrated through the following simple examples.

Assume that 20 tensile specimens of 2219 aluminum produced a mean ultimate stress of 68 ksi with a coefficient of variation of 0.024 and a mean yield stress of 55 ksi with a coefficient of variation of 0.027. One-sided K factors ($n_R=K$) for 20 specimens are 3.3 and 2.4 for A basis and B basis,[4] respectively. The lean safety factor calculated from Eqs. (2), (3a), and (5) is approximately 1.25 for both bases. The combined applied stress coefficient of variation is 0.15, and the load modeling uncertainty is assumed as 0.10. Combining these two coefficients by the root-sum-squared rule of variances, the applied stress lumped coefficient of variations is 0.18. The applied mean stress is calculated from Eq. (6) for each example. The operational probability range is based on the probability range factor of the applied stress, where factors of two and three represent probabilities of 0.995 and 0.997, respectively. Results from varying the three safety design factors are listed in Table 1.

Example 1 is a baseline case that incorporates the lean safety factor defined by Eq. (5), a B-basis material having probability range factor K=2.4, and a 2-sigma applied stress that is allowed to exceed the limit stress once in 200 applications. The resulting ultimate structural reliability is 0.9_5, or a failure rate of 1 in 100,000. It is recognized that reliabilities defined by Eqs. (7) and (8) are first-order approximations based solely on means and standard deviations and not on the real content of very long tails. Therefore, reliabilities above 0.9_4 would seem to overextend the math model sensitivity.[5] Uncertainties in statistical data would also seem to require structures to be designed to 0.9_6 to guarantee 0.9_4 reliability.

Example 2 uses an A-basis material but again allows the applied load to exceed the yield stress once in 200 uses. Example 3 allows one yield stress exceedance in 300 uses. In both examples, increasing the probability range factors by approximately 15% increases the ultimate reliability one order of magnitude.

Example 4 is typical of current practices. Although the calculated lean safety factor is 1.25, the arbitrarily selected conventional safety factor of 1.4 is excessive, which spills into the applied stress probability range and effectively increases the probability range factor. This interchange may be quantified by using Eq. (6) with the lean safety factor, and again with an arbitrarily selected safety factor, and solving simultaneously. The resulting effective applied stress probability range factor is

$$\tilde{n}_A = \frac{SF}{SF_{LL}} \frac{(1+n_A C_A)}{C_A} - \frac{1}{C_A} \tag{9}$$

Applying example 4 to Eq. (9) increases the effective applied stress probability from a 3σ to a 4σ and the ultimate reliability three orders of magnitude. Example 5 is identical to example 4 except that the lean safety factor of the material is 1.5, which is greater than the specified conventional safety factor. The resulting applied stress probability range is reduced to a marginal 3σ, though the ultimate stress reliability is unchanged.

Implied by the derived applied mean stresses in all examples is that increasing the conventional safety factor decreases the applied stress limit and allots less of the available elastic property of the material for operational design loads. Consequently, increasing the conventional safety factor decreases the allowed limit load, which increases the structural thickness and weight. Therefore, weight and the subsequent cost of it may be considered as the upper bound design driver of the conventional safety factor. Substituting thickness (or area) dimensions of common stress equations into a weight equation of unit length provides the linear relation of structural weight increase with safety factor in excess of the lean factor,

$$\frac{\partial w}{w} = m\frac{\partial SF}{SF} \tag{10}$$

The weight coefficient is unity for plate normal stresses and 0.5 for bending. Structural weight increase is further perpetrated through weight and cost increases in manufacturing, handling, spares, propulsion system, propellant, and payload.

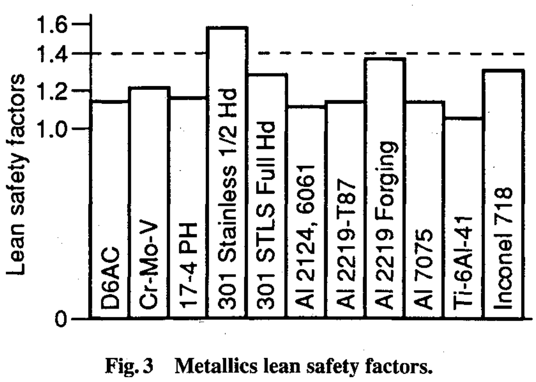

Fig. 3 Metallics lean safety factors.

Several significant observations may be drawn from this exercise. First-order reliability parameters are equally applicable to conventional safety factor methods. Safety design factors must satisfy loads, stress, and material requirements. The net structural safety of combined safety design factors may be expressed as a safety index or reliability. Metallic materials exhibit dual failure modes: yield and ultimate. Specifying arbitrary ultimate conventional safety factors, or reliability levels, with no consideration of the lean safety factor may compromise the structural performance under design operational loads. Applying a current universal safety factor on aluminum structures and welds will result in a much lower yield or operational margin than would be produced with high-strength steels.

Operational Reliability

Because large ultimate safety factors do not necessarily guarantee adequate yield margins as shown by Eq. (9), a yield safety index determination is pertinent to a complete safety analysis. Equating the applied stress upper tolerance limit with the resistive stress lower tolerance limit about the yield stress produces

$$F_{ty} = \mu_A (1+\tilde{n}_A C_A) = \mu_{ty}(1-n_R C_{ty})$$

and substituting both mean stresses into Eq. (6) gives

$$\mu_{ty} - \mu_A = \mu_A (\tilde{n}_A C_A + n_R C_{ty}) / (1-n_R C_{ty})$$

Substituting the difference of the means into Eq. (7) provides the desired yield safety index in terms of the effective applied stress range factor,

$$Z_{ty} = \frac{\tilde{n}_A C_A + n_R C_{ty}}{(1-n_R C_{ty})\,[C_A^2 + (C_{ty}\mu_{ty}/\mu_A)^2]^{1/2}} \tag{11}$$

Its reliability is determined from Fig. 2. Equation (11) was applied to examples in Table 1, which illustrate the sensitivity of the yield safety index with variations in safety design factors.

Selection Criteria

As observed in the preceding section, the deterministic method of Eq. (1), expressed by a nonarbitrarily selected probabilistic parameter in Eqs. (2) and (3a), should provide a safe and uniformly efficient structure. Selection criteria of safety design parameters must be based on the relative risk, which is defined as the product of the probability of failure and cost consequence of the failure. The safety technique must include estimates of modeling and complexity uncertainty parameters.

Resistive Stress

Material properties are generally specified by A or B basis, both of which incorporate a K factor in lieu of the probability range factor to adjust for the probability and confidence of the sample size. The A basis should apply to primary structures having high risk. A material uncertainty factor should also be included to account for material aging, new application, residual stresses, etc., and may be lumped together with the conventional safety factor for simplicity.

Conventional Safety Factor

The conventional safety factor is a composite of the lean safety factor and the material uncertainty factor discussed earlier. It must also include fabrication and stress modeling uncertainties that are represented by probability distributions estimated from experience and test data base. A complexity, or judgment, factor is added for design oversights, alternate load path, joints, compound discontinuities, etc., and uncertainties on highly redundant and complex structures. Although these uncertainty elements must be incorporated in all design analyses, their purpose is to be sacrificed during tests as required by the hardware without reducing the lean safety factor. Only the lean safety factor must be guaranteed and verified.

The lean safety factor defined by Eq. (5) was determined to be the lower bound of the inelastic region of a crystalline material. The inelastic region is perceived as a safety bonus that allows a sneak load path or an unpredicted, onetime malfunction load to yield and deform the structure in a safe mode without having to penalize the linear stress region reserved for operational design loads. A lower lean safety factor would allow a predicted operational stress to permanently deform the metallic structure. A higher value would unnecessarily increase its weight. Figure 3 illustrates variations in the lean safety factor over a range of contemporary metallic materials.

Applied Stress

The applied stress distribution is a composite of von Mises stresses induced by dynamic and static load distributions. Probability range factors are selected at each induced stress region based on risk of structural failure. Induced flight loads produce the greatest variance, are the most unpredictable, and represent the greatest risk. Static loads may require lower factors; however, pressure vessels must consider large probability range factors based on the risk of their energy content. The applied stress must also include a loads modeling uncertainty as a variance that may be combined with the composite applied stress variance. The verification applied load must include loads modeling uncertainty variances.

Split Stress Factor

More efficient stress designs of very large aerostructures may be realized by separating the applied stress into one induced by well-defined loads (pressure, thrust, and inertia) and another induced by less certain loads (aerodynamics, dynamics, and winds). The maximum allowed applied stress is then the sum of the split applied stresses,

$$F_A = F_{A1} + F_{A2} \tag{12}$$

Applied stress components are defined in upper tolerance limit format and as rule of mixtures fractions,

$$F_{A1} = \mu_{A1}(1 + n_{A1}C_{A1}) = qF_A \tag{13}$$

$$F_{A2} = \mu_{A2}(1 + n_{A2}C_{A2}) = (1-q)F_A \tag{14}$$

Distribution parameters in Eqs. (13) and (14) are combined into Eq. (12) through the basic rules of statistics for combining uncorrelated variables. The mean of the overall distribution is the algebraic sum of the means, and the combined variance is the sum of the variances,

$$\sigma_A^2 = \sum_{i=1}^{N} (\mu_{Ai}C_{Ai})^2 \tag{15}$$

Solving for the means from Eqs. (13) and (14),

$$\mu_{A1} = \frac{qF_A}{(1 + n_{A1}C_{A1})}, \quad \mu_{A2} = \frac{(1-q)F_A}{(1 + n_{A2}C_{A2})}$$

and summing gives the combined mean stress

$$\mu_A = F_A\left[\frac{q}{1 + n_{A1}C_{A1}} + \frac{(1-q)}{1 + n_{A2}C_{A2}}\right] \tag{16}$$

Solving for the product of the coefficient of variation and probability range factor from Eqs. (13) and (14) gives

$$n_{A1}C_{A1} = \frac{qF_A}{\mu_{A1}} - 1 \tag{17a}$$

$$n_{A2}C_{A2} = \frac{(1-q)F_A}{\mu_{A2}} - 1 \tag{17b}$$

Treating the product squared as a variance and combining in Eq. (15) gives

$$(n_AC_A)^2 = (n_{A1}C_{A1})^2 + (n_{A2}C_{A2})^2 \tag{18}$$

Substituting Eq. (17) into (18) yields the desired combined product of the coefficient of variation and probability range factor,

$$n_AC_A = F_A\left[\left(\frac{q}{\mu_{A1}} - 1\right)^2 + \left(\frac{(1-q)}{\mu_{A2}} - 1\right)^2\right]^{1/2} \tag{19}$$

A similar approach may be used to combine different types of loads, stresses, and manufacturing distributions.

Conclusions and Remarks

The deterministic safety concept is still the most versatile design method, and it is verifiable. It consists of three safety design factors that are independently specified by loads, stress, and materials. Their combined effects establish the relative safety of the structure through the safety index and reliability.

Prevailing interest in the conventional safety factor has been partially attributed to significant progress in loads analyses, material properties, and simplicity of the conventional safety factor. However, current universal safety factors based on ultimate stress bias the operational deformation margins by the metallic material selection. Materials with larger ultimate-to-yield stress ratios provide lower yield margins. This bias may be overcome by modifying the conventional safety factor to consist of the ultimate-to-yield stress ratio lower limit and all of the modeling and complexity uncertainty factors. Conventional safety factors on nonmetallic materials may be similarly modified, emphasizing material variation factors related to processing and unique operational conditions in the absence of the yield limit.

The primary purpose of metallic structures is to sustain operational loads without detrimental deformation. There is no similar compelling physical condition on ultimate safety, nor is there a single universal safety factor that may provide uniformly efficient aerostructure.

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Earl A. Thornton
Associate Editor